Homework 5

518021911160 窦嘉伟

Problem 1

1. ¬(∃x)(∃y)(P(x)∧P(y)∧Q(x)∧Q(y)∧R(x, y)) =(∀x)(∀y)((P(x)∧P(y)∧Q(x)∧Q(y))→ ¬R(x, y))

证明:

¬(∃x)(∃y)(P(x)∧P(y)∧Q(x)∧Q(y)∧R(x, y))

= (∀x)(∀y)¬(P(x)∧P(y)∧Q(x)∧Q(y)∧R(x, y))

= (∀x)(∀y)(¬(P(x)∧P(y)∧Q(x)∧Q(y))∨¬(R(x, y)))

=(∀x)(∀y)((P(x)∧P(y)∧Q(x)∧Q(y))→ ¬R(x, y))

1. (∃x)(P(x) → Q(x)) =(∀x)P(x) → (∃x)Q(x)

证明:

(∃x)(P(x) → Q(x))

=(∃x)(¬P(x) ∨ Q(x))

=(∃x)¬P(x) ∨(∃x) Q(x)

=¬(∀x)P(x) ∨(∃x) Q(x)

=(∀x)P(x) → (∃x)Q(x)

1. (∀y)(∃x)((P(x) → q) ∨ S(y)) =((∀x)P(x) → q) ∨ (∀y)S(y)

(∀y)(∃x)((P(x) → q) ∨ S(y))

=(∀y)( (∃x)(P(x) → q) ∨ S(y) )

=(∀y)( ((∀x)P(x) → q) ∨ S(y) )

=((∀x)P(x) → q) ∨ (∀y)S(y)

1. (∃x)P(x) → (∀x)Q(x) ⇒(∀x)(P(x) → Q(x))

(∃x)P(x) → (∀x)Q(x)

=¬(∃x)P(x) ∨ (∀x)Q(x)

=(∀x)¬P(x) ∨ (∀y)Q(y)

⇒ (∀x)(P(x) → Q(x))

Problem 2

(∀x)(P(x) ∨ Q(x)) ∧ (∀x)(Q(x) → ¬R(x)) ⇒ (∃x)(R(x) → P(x))

1. (∀x)(P(x) ∨ Q(x)) 前提
2. ¬P(x)→ Q(x) 全称量词消去
3. (∀x)(Q(x) → ¬R(x)) 前提
4. Q(x) → ¬R(x) 全称量词消去
5. ¬P(x)→¬R(x) 24分离
6. R(x)→P(x) 5置换
7. (∃x)(R(x) → P(x)) 存在量词引入

Every student in the university is either an undergraduate or a postgraduate. Some students are male. John is not a postgraduate but he is male. Therefore, if John is a student in the university, he must be an undergraduate. Represent these statements in predicate logic and prove the conclusion (”if John is a student in the university, he must be an undergraduate”) by resolution method in 5.6

P(x) means x is an undergraduate,Q(x) means x is a postgraduate ,M(x)means x is male and U(x) means x is in the university.

(∀x)( U(x)→(P(x) ∨ Q(x)) )

(∃x) M(x)

¬Q(John) ∧ M(John)

U(John) → P(John)

证明

U(John) → P(John)可通过证明 (∀x)( U(x)→(P(x) ∨ Q(x)) )∧ ¬Q(John)→ (U(John) → P(John))

令 G = (∀x)( U(x)→(P(x) ∨ Q(x)) )∧ ¬Q(John) ∧¬(U(John) → P(John))

= (∀x)( ¬U(x) ∨ P(x) ∨ Q(x) )∧ ¬Q(John)∧(U(John) ∧ ¬P(John) )

子句集S= {¬U(x) ∨ P(x) ∨ Q(x),U(John) , ¬P(John),¬Q(John) }

1. ¬U(x) ∨ P(x) ∨ Q(x)
2. U(John)
3. ¬P(John)
4. ¬Q(John)
5. P(John) ∨ Q(John) 12归结
6. Q(John) 34归结
7. 46归结

Problem 3

1 wrong

如果能，有( (∀x)P(x) → Q(x) )⇒( P(a) → Q(a) )

令G = ( (∀x)P(x) → Q(x) ) ∧( P(a) ∧¬Q(a) )

= (¬P(b) ∨Q(x) )∧( P(a) ∧¬Q(a) )

S = {(¬P(b) ∨Q(x) ), P(a) , ¬Q(a) }

无法归结

2 wrong

∃x无法作用于∧

3同上

Problem 4

1 universally valid

((∃x)P(x) → (∃x)Q(x)) → (∃x)(P(x) → Q(x))

令G =( (∀x)¬P(x)∨(∃x)Q(x) ) ∧ (∀x)(P(x) ∧¬Q(x))

S = {¬P(x)∨Q(a),P(x),Q(x)}

归结得

2 not ex: P(x,y)表示x+y>0

Problem 5

Unsatisfiable

Problem 6

f(b) = b, f(b) = f(a), a = c, b != c, f(c) != a

V1 = f(b) V2=f(a) V3=f(c)

V3

a,c

V1,b ,V2

a,c

V1,b ,V2,V3

Satisfiable

Problem 7

f(i) − f(j) != 0 ∧ i − j = 0

m-n!=0 ∧ i − j = 0 ∧ m=f(i) ∧ n=f(j)

m-n!=0 ∧ i − j = 0 m=f(i) ∧ n=f(j)

Arithmetic Solver EUF

i=j与i!=j时，左边都unsat,最终返回unsat